VI. Discussion

The main result of this investigation can be stated as follows. If the rate of twist is relatively low and the anisotropy relatively high, then the field can be thought of as attaching itself to the structure of the medium as it propagates through it, and it therefore rotates. This situation corresponds to a low value of v and the conclusion stated above is evident from (34) due to the relative unimportance of the cross terms of the transfer matrix. In general, however, the twist generates polarization coupling and an alteration of propagation constants.

Discontinuities perpendicular to the direction of propagation can be handled. Computer calculation would generally be necessary, but marked simplification occurs in special cases notably for a taper region for which explicit formulas can be obtained.

A physical structure having the properties dealt with in this paper is not hard to visualize. It could consist of layers of a fabric in which warp and weft have markedly different dielectric properties, each layer being oriented at an angle with respect to the adjacent layer. Interesting speculation on this matter is contained in a recent letter by Shelton.[3] Any degree of twist per unit wavelength is possible with these structures but the range of anisotropy appears to be limited. A polarizer having a modest improvement in frequency bandwidth (corresponding to $v^2 = 1/2$) is feasible. The frequency independent-relations (39) and (40) can also be realized, but with a low anisotropy interesting polarization properties would require an excessive thickness of material.

Finally it should be pointed out that this paper deals with a one-dimensional problem. The lateral limitation of the geometry by means of a waveguide or other boundary would greatly complicate it.

[3] P. Shelton, "Comments on 'polarization transformation in twisted anisotropic media,'" *IEEE Trans. Microwave Theory and Techniques* (*Correspondence*), vol. MTT-14, p. 579, November 1966.

The Numerical Solution of Rectangular Waveguide Junctions and Discontinuities of Arbitrary Cross Section

CORNELIS A. MUILWYK, STUDENT MEMBER, IEEE, AND J. B. DAVIES

Abstract—**A method is described of calculating automatically the performance of junctions of rectangular waveguides including conducting cylinders of arbitrary shape. The only restriction is that the overall problem should be effectively two-dimensional, i.e., the structure be uniform in some cross section. The one basic approximation made (which could be removed) is shown to give useful results for the devices tested, viz., for various shaped irises (inductive and capacitive) and the 4-port *H*-plane junction.**

I. Introduction

In an earlier paper [1], the authors described a method of solving the problem of the hollow waveguide of arbitrary shape, and indicated that the procedure could be applied directly to the solution of a wide range of waveguide discontinuity problems of engineering interest. The object of this paper is to describe the application and to give some typical results.

Manuscript received December 5, 1966; revised March 23, 1967. The work of C. A. Muilwyk was supported by the Athlone Fellowships Managing Committee and the National Research Council of Canada.

C. A. Muilwyk is with General Staff Engineering, Alberta Government Telephones, Edmonton, Canada. He was formerly at the Dept. of Elec. Engrg., the University of Sheffield.

J. B. Davies is with the Dept. of Elec. Engrg., the University of Sheffield, Sheffield, England.

As with the previous paper [1], the object is to enable a wide class of problems to be solvable with the one method *and the one computer program*. It should be emphasized that the technique of this paper depends on being able to calculate the cutoff frequencies of an arbitrarily shaped waveguide. Other methods have been described [2], [3] besides that used in this paper, but it is not clear from published results whether any of these is as automatic and rapid in computing.

The method can be applied directly to the analysis of a 2-, 3-, or m-port junction of rectangular waveguides containing arbitrarily shaped conducting structures. The waveguides may have different dimensions, but the overall structure must be uniform (i.e., have constant cross section) in one direction (either the "broad" or "narrow" transverse direction) so that the resulting boundary-value problem is effectively two-dimensional. Examples of such structures would include the conducting post or iris (of any shaped cross section) in rectangular waveguide, an offset or change of transverse dimension in the rectangular waveguide, and for m-port junctions the T, Y, and 4-port cross junctions. All these examples could be in the E plane or H plane.

The method used relies on analysis of the junction when supporting pure standing waves, as used experimentally in the "nodal-shift" or Weissfloch-Feenberg method of mea-

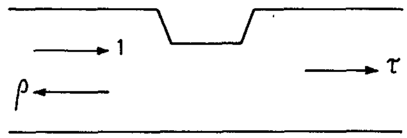


Fig. 1. Typical rectangular waveguide with symmetrical discontinuity.

surement [4] for a loss-free junction. The results given involve just one basic approximation, corresponding to taking a measurement with the short circuit and null detector close to the junction. Results are obtained for a variety of geometries, to verify the procedure, and are presented here. The method could be extended to avoid the basic approximation.

II. Theory

To simplify the presentation and discussion of the analysis, we shall restrict ourselves to rectangular waveguides containing discontinuities that are physically symmetrical about the central transverse plane. The application to junctions without this symmetry, or with more than two ports, should be apparent later.

Our typical structure is shown in Fig. 1, and can represent a cross section in either the E or H plane. The scattering matrix is

$$\begin{pmatrix} \rho & \tau \\ \tau & \rho \end{pmatrix},$$

and the reflection and transmission coefficients can be described [5] in terms of the eigenvalues λ_1 and λ_2 of the scattering matrix by

$$\rho = \tfrac{1}{2}(\lambda_1 + \lambda_2) = \tfrac{1}{2}(\exp j\theta_1 + \exp j\theta_2) \qquad (1)$$

$$\tau = \tfrac{1}{2}(\lambda_1 - \lambda_2) = \tfrac{1}{2}(\exp j\theta_1 - \exp j\theta_2). \qquad (2)$$

The junction is presumed loss free, so that the eigenvalues must lie on the unit circle, and the junction is described completely by two real numbers θ_1 and θ_2. The eigenvectors (1, 1) and (1, −1) correspond to pure standing waves in the junction that are either symmetric or antisymmetric about the junction center plane [5]. Figure 2 shows these standing-wave patterns, giving the magnetic fields for an H plane discontinuity. Similar patterns are shown in Fig. 3 of the electric fields for an E plane discontinuity. If, either by field theory or by experiment, we can find the positions of these two sets of standing waves, we will know the eigenvalues, and hence by (1) and (2) the elements, of the scattering matrix. Specifically we require the positions of two electric nulls that can be established equidistant from the junction, but sufficiently distant from the junction for evanescent fields to be negligible.

For the H plane discontinuity of Fig. 2, there is no field variation normal to the H plane; for the E plane discontinuity of Fig. 3 there is a known variation [$\sin(\pi x/x_0)$] normal to the E plane. The three-dimensional standing waves of Fig. 2 or 3 are equivalent, then, to the fields of a suitable

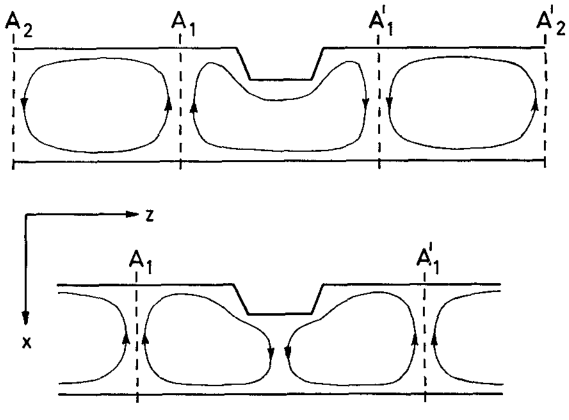


Fig. 2. Magnetic fields for the even and odd standing waves of an H plane discontinuity in rectangular waveguide.

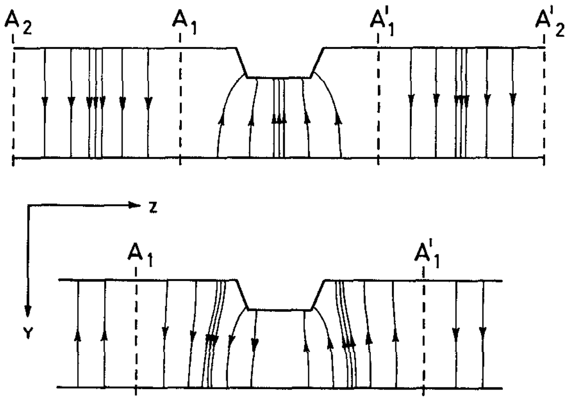


Fig. 3. Electric fields for the even and odd standing waves of an E plane discontinuity in rectangular waveguide.

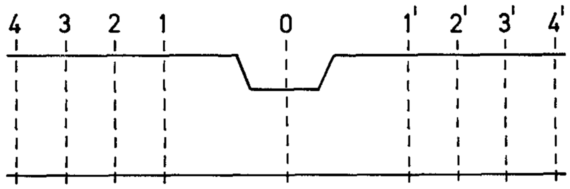


Fig. 4. Waveguide cross section of "equivalent problem," with electric walls at 1 and 1′, 2 and 2′, etc.

mode present in a uniform ridged waveguide of cross section as shown in Fig. 4. For an H plane discontinuity, with magnetic fields as in Fig. 2, and with short circuits at A_1, A_1', the fields are identical to a TM mode of the ridged guide in Fig. 4, at cutoff. For an E plane discontinuity, the fields are identical to a TE mode of the same ridged guide, operating at the frequency for which its guide wavelength equals the cutoff wavelength of our original rectangular waveguide.

In either case we have the two-dimensional boundary-value problem of finding the wall positions (A_1 and A_1', or for more accuracy A_n and A_n') that support the particular wave at a particular frequency. The converse problem, of finding the cutoff frequency for any particular wall or short-

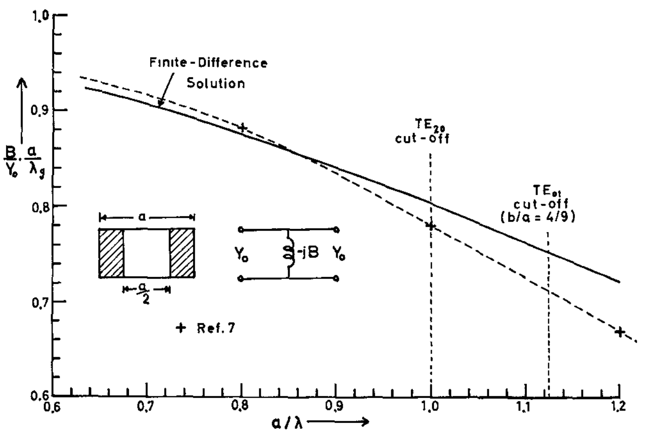


Fig. 5. Shunt susceptance of a thin inductive iris, with finite difference and approximate analytic solutions.

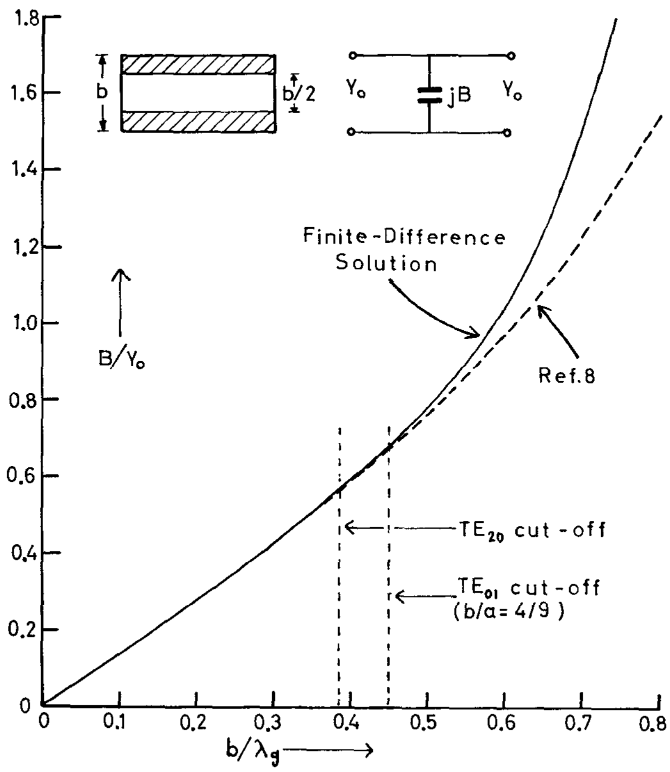


Fig. 6. Shunt susceptance of a thin capacitive iris, with finite difference and approximate analytic solutions.

circuit position, is precisely that solved by the authors [1]. In this method, the dominant TE or TM mode of an arbitrary shaped waveguide is solved automatically, simply by prescribing the shape geometrically, via a few lines of Autocode instruction, to a computer program.

The procedure used to solve any given waveguide structure is as follows. The cutoff wavelengths of the appropriate TE or TM mode are calculated for a number of cross sections, bounded by the electric walls 1, 1′; 2, 2′; etc. Each solution gives a wavelength at which the standing wave pattern would be established, and hence gives one eigenvalue for the scattering matrix at this same operating wavelength. From results of calculations of a range of cross sections, with short circuits at various distances from the junction, we obtain the two eigenvalues, and hence a complete description of the junction, over a band of frequencies.

Because the computer program referred to solves only for the dominant TE or TM mode, we are limited to analyzing our waveguide junction with the electric nulls nearest to the discontinuity. This introduces the only basic approximation in the present procedure. It could be removed by generalizing the computer program to deal with higher modes in the waveguide. However, as will be seen in the next section, quite useful results are obtained even with this limitation.

For a junction with the physical symmetry assumed in this section, we can economize in our calculation. The computer program for the calculation of the arbitrarily shaped waveguide can "mix" the boundary conditions [6], to simulate the electric or magnetic wall of any symmetry plane, and so calculate separately the symmetric and antisymmetric modes. This means that in the above procedure it is necessary to consider only one half or side of the symmetric junction—from 0 to 1, 2, 3, etc. of Fig. 4. The calculations with electric and magnetic walls at the symmetry plane will result in an associated pair of eigenvalues $\exp(j\theta_1)$ and $\exp(j\theta_2)$, and hence the scattering matrix, at some frequency.

The analysis of a nonsymmetric 2-port junction would apply in the same way, just as the nodal-shift experiment would apply. However, the simplicity of description via a scattering matrix with known eigenvectors is lost, as is the advantage of having to analyze only one half of the junction.

For simplicity, the preceding theory has been presented for a 2-port device. The procedure can, however, apply equally to m-port devices (within the same geometric restrictions of shape given in the Introduction). This application is illustrated in the next section, when considering a 4-port H-plane junction.

III. Results

In order to verify the theory and to obtain some estimate of the accuracy which may be achieved, a number of problems have been solved. In all cases the results obtained using the finite-difference technique have been compared with other existing theories or experimental data.

A. Thin Irises

The first type of discontinuity to be considered was that of thin irises in a rectangular waveguide. Figure 5 shows the equivalent circuit parameter as a function of frequency for two symmetrical inductive irises. The points shown for comparison were taken from Marcuvitz [7], who estimates the accuracy of his results to be within 1 percent for $a/\lambda < 1$, but greater than 1 percent for $a/\lambda > 1$. Our results agree to within about 2 percent for $a/\lambda < 1$. The susceptance of two symmetrical capacitive irises was calculated next, and the results, compared with those of Montgomery, Dicke, and Purcell [8], are plotted in Fig. 6. The results for both types of thin irises show good agreement for frequencies within the usual range of interest, i.e., for frequencies below the cutoff

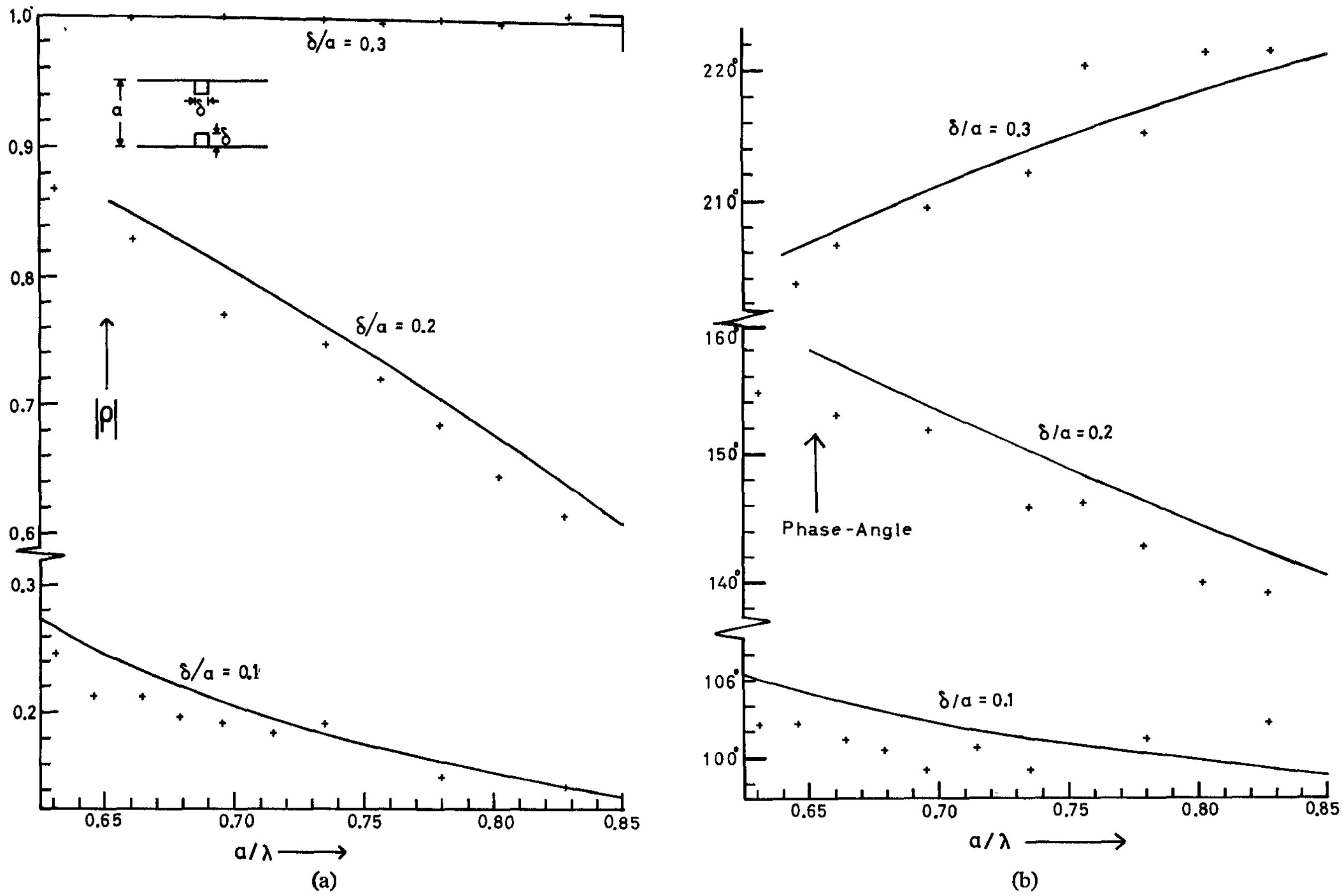


Fig. 7. Magnitude and phase of reflection coefficient of square inductive irises (discontinuities in the H plane) with results of theory and experiment.

frequency of the next higher mode. For higher frequencies the discrepancy increases as expected, because the short circuit has to be placed closer to the discontinuity and the error due to our fundamental approximation becomes increasingly significant.

B. Thick Irises

Secondly, we considered two types of thick irises in rectangular waveguides. Figure 7(a) and (b) shows the magnitude and phase angle of the reflection coefficient due to square inductive irises of various sizes. The experimental points were obtained using the nodal-shift method, but examining just the two standing wave situations of Fig. 2. The symmetrically placed short circuit and standing-wave indicator probe then give the scattering matrix eigenvalues directly. Although the agreement of Fig. 7(a) and (b) may at first glance not appear very good, it should be noted that a fairly expanded scale has been used and that in most cases the finite-difference results are straddled by the experimental points. In general, the discrepancy between the calculated curve and the average of the experimental data is no worse than 2 or 3 percent.

Figure 8 shows the voltage standing-wave ratio (VSWR) as a function of frequency due to a single half-round inductive obstacle with $R/a=2/9$. The curve shown for comparison is an approximately linear interpolation between two curves given by Kerns [9] for $R/a=0.20$ and $R/a=0.24$. Again the two results agree to within a few percent. In particular, for $a/\lambda=0.7162$, Kerns obtains an accurate value of 2.1125 for VSWR, whereas the finite-difference calculation resulted in a value of 2.15, an error of 1.7 percent.

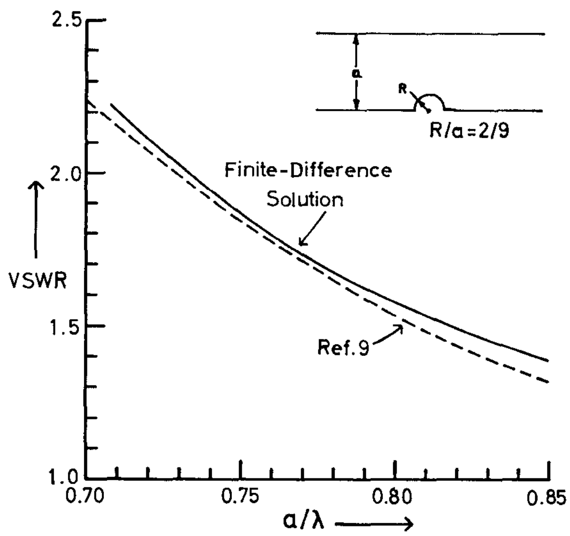
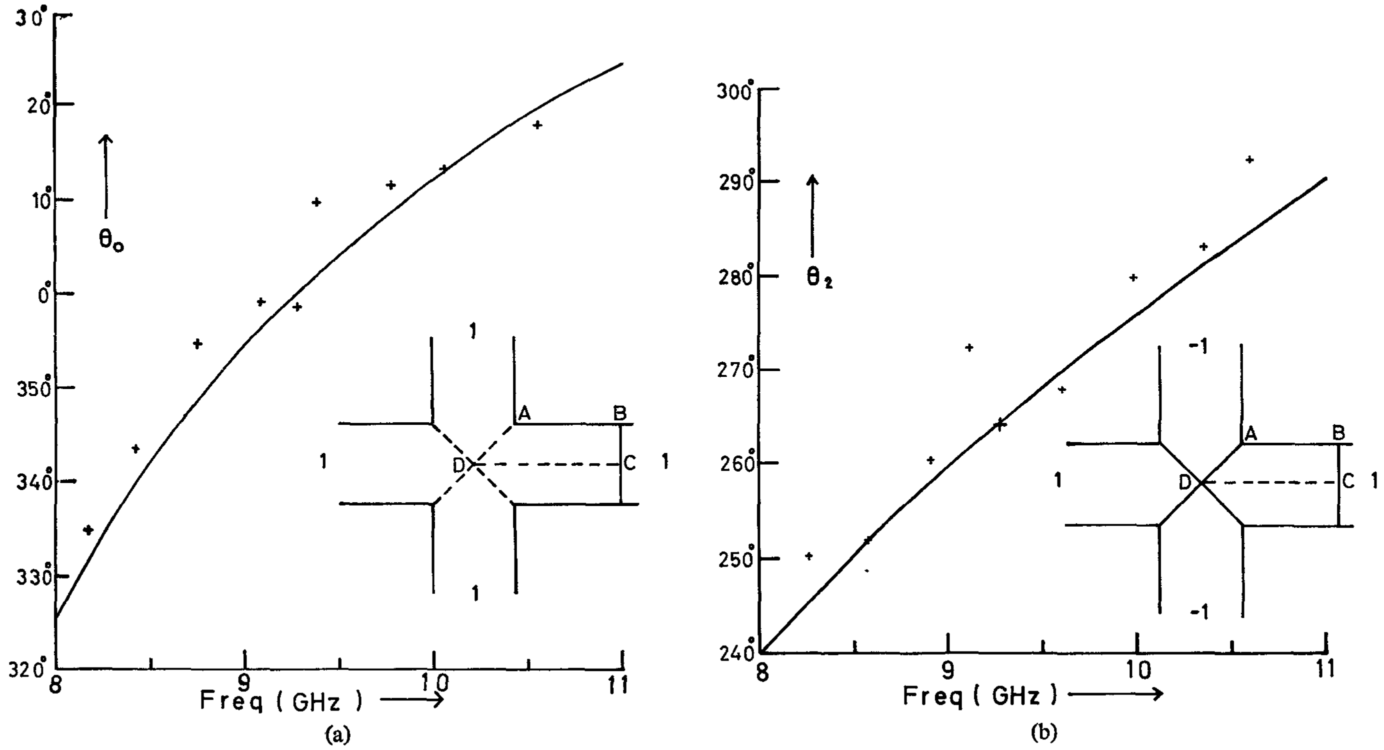


Fig. 8. Single half-round inductive obstacle, by finite difference and by exact analytic methods.



C. The 4-Port H-Plane Junction

Finally, to illustrate the application to many-port junctions, we calculated the scattering matrix eigenvalues for the 4-port H-plane junction of rectangular waveguides. The results are given in Fig. 9(a), (b), (c), and again show very good agreement with the experimental results given by Davies [10]. It might be interesting to note here that for $\theta_0 > 0°$ in Fig. 9(a), it was necessary to place the short circuits inside the junction and thus to find the cutoff frequency of the TM_{11} mode in the resulting square waveguide. Even so, the results for this range show excellent agreement with experiment.

Because of the 90° rotation symmetry of the junction, it is possible to calculate separately each eigenvalue, by considering just one short-circuited waveguide. By calculating the waveguide with electric walls along the diagonal symmetry lines [see inset to Fig. 9(b)], we obtain the eigenvalue $\exp(j\theta_2)$ with eigenvector $(1, -1, 1, -1)$. Similarly, we obtain the other eigenvalues from magnetic walls [Fig. 9(a)] or magnetic and electric walls [Fig. 9(c)] along the symmetry lines. In each case, the economized shape to be calculated on the computer is ABCD(E).

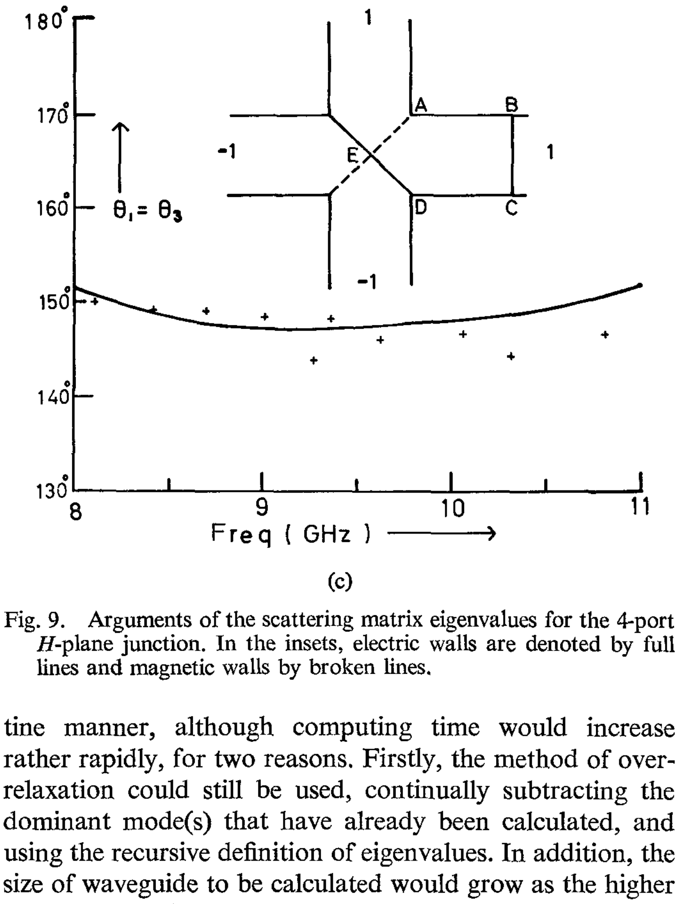


Fig. 9. Arguments of the scattering matrix eigenvalues for the 4-port H-plane junction. In the insets, electric walls are denoted by full lines and magnetic walls by broken lines.

IV. Conclusions

It has been shown that useful results can be calculated *in a routine manner* for a wide range of waveguide discontinuity and junction problems. The errors, which were rarely greater than 3 percent for the examples calculated, could clearly be reduced by calculating modes above the dominant in the related waveguide problem. This could be done by the finite-difference method, still in an automatic and routine manner, although computing time would increase rather rapidly, for two reasons. Firstly, the method of overrelaxation could still be used, continually subtracting the dominant mode(s) that have already been calculated, and using the recursive definition of eigenvalues. In addition, the size of waveguide to be calculated would grow as the higher accuracy is being sought, corresponding to the short circuits being a number of half-guide wavelengths from the junction. For these reasons, high accuracy would be expensive in computing time. This is, however, something of an open question as so few structures permit alternative, analytic solutions beyond first-order perturbation.

This method is therefore proffered as a versatile and automatic procedure for analyzing, with moderate accuracy, this class of waveguide problems.

ACKNOWLEDGMENT

The authors are indebted to S. J. Liebmann for the experimental results of Fig. 7(a) and (b).

REFERENCES

[1] J. B. Davies and C. A. Muilwyk, "Numerical solution of uniform hollow waveguides with boundaries of arbitrary shape," *Proc. IEE* (London), vol. 113, pp. 277–284, February 1966.
[2] H. Y. Yee and N. F. Audeh, "Uniform waveguides with arbitrary cross section considered by the point-matching method," *IEEE Trans. Microwave Theory and Techniques*, vol. MTT-13, pp. 847–851, November 1965.
[3] P. A. Laura, "Conformal mapping and the determination of cutoff frequencies of waveguides with arbitrary cross section," *Proc. IEEE (Letters)*, vol. 54, pp. 1078–1080, August 1966. (This letter includes further references.)
[4] N. Marcuvitz, *Waveguide Handbook*. New York: McGraw-Hill, 1951, p. 132 et seq.
[5] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, *Principles of Microwave Circuits*. New York: McGraw-Hill, 1948, ch. 12.
[6] J. B. Davies and C. A. Muilwyk [1], sec. 2.6.
[7] N. Marcuvitz [4], pp. 221–224.
[8] C. G. Montgomery, R. H. Dicke, and E. M. Purcell [5], pp. 166–167.
[9] D. M. Kerns, "Half-round inductive obstacles in rectangular waveguide," *J. Research NBS*, vol. 64B, pp. 113–130, April–June 1960.
[10] J. B. Davies, "An analysis of the *m*-port symmetrical *H*-plane waveguide junction with central ferrite post," *IRE Trans. Microwave Theory and Techniques*, vol. MTT-10, pp. 596–604, November 1962.

The Green's Dyadic for Radiation in a Bounded Simple Moving Medium

Y. J. SETO, MEMBER, IEEE

Abstract—**The studies here show that the wave equation for electromagnetic wave propagation in an isotropic and uniformly moving medium is solvable by the separation method in four coordinate systems. Solutions in the form of complete sets of eigenfunctions are possible for problems where boundary surfaces are presented. A Green's dyadic for finite or semi-infinite domain problems involving sources in the moving medium has been formulated through vector operation on the eigenfunction solutions of the homogeneous wave equation. The case of electromagnetic waves excited by a current loop, immersed in a moving medium, and confined by a circular cylindrical waveguide, was examined. The electric and magnetic field intensities in such a waveguide were compared with those obtained through a different approach. The Green's dyadic for electromagnetic waves in an infinite domain moving medium was shown to be obtainable from the finite domain Green's dyadic through a limiting process.**

INTRODUCTION

THE PROBLEM OF electromagnetic wave propagation in a moving medium has gained a renewed interest in recent years. A number of studies has been reported on the subject involving a bounded or an unbounded moving medium. For radiation problems, Lee and Papas[1] have derived a Green's function which is adequate for sources in an infinite domain moving medium. Compton and Tai[2] also have derived an infinite domain Green's dyadic for sources in a moving medium which has a different form from that obtained by Lee and Papas. In principle, the infinite domain Green's function can be used to obtain the field in a finite domain if one retains the surface integral in the integral representation of the field. In practice, however, evaluation of the surface integral is not a simple task. For most boundary value problems involving sources inside the boundaries, the boundary conditions are usually either homogeneous Dirichlet or homogeneous Neumann, and seldom involve both homogeneous Dirichlet and homogeneous Neumann simultaneously on the same boundary surface. Any inhomogeneous boundary condition requires a priori knowledge of the surface charge density or surface current density before the surface integral can be evaluated. Such knowledge is usually not given in the statements of the problem.

To avoid such difficulties, a different approach is suggested in this paper. A study to better understand the finite or semi-infinite domain free-wave solutions is carried out.

Manuscript received December 12, 1966; revised April 17, 1967. This work was supported in part by the National Science Foundation under Grant GK-882. Part of this work was carried out during this author's tenure at the University of Houston, Houston, Tex.

The author is with the Dept. of Elec. Engrg., Tulane University, New Orleans, La. 70118

[1] K. S. H. Lee and C. H. Papas, "Electromagnetic radiation in the presence of simple moving medium," *J. Math. Phys.*, vol. 5, no. 12, pp. 1668–1672, 1964.

[2] R. T. Compton, Jr., and C. T. Tai, "Radiation from harmonic sources in a uniformly moving medium," *IEEE Trans. Antennas and Propagation*, vol. AP-13, pp. 574–577, July 1965.